

## Errors analysis solving problems analogies by Newman procedure using analogical reasoning

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### Abstract

Errors in solving mathematical problems often done by students. For see the types of errors that occur need to analyzed by Newman procedures. The purpose of this study to analyze the errors that occurred in the students in solving problems analogies using procedure Newman. Research using qualitative research methods and techniques of data collection using interviews. Subjects of research were 148 high school students in West South Nusa, Indonesia. The instrument used in the research is problems analogies contains two problems: the source and the target. The students are required to have the ability to associate the problems encountered by the previous problem, because mathematical concepts are connected. Students in problem solving targets need to do something. Students will begin to read and understand the problem. Students will determine the exact formula for the problems to be tackled by linking the problems encountered with previous problems that they already know the solution. Furthermore, students perform arithmetic operations and get the solution of the problem. Students can make mistakes in solving problems. The results showed errors types by Newman procedures and additional errors from student carelessness. Newman procedure, can see the mistakes made by students in solving the problems analogies so as to provide an overview to the teachers to develop learning involving analogy problems as tasks/exercises and exams.

**Keywords:** Errors, analysis, problems analogies, procedure Newman, analogical reasoning

### Introduction

#### Problems Analogies

Analogy is composed of three types: classical analogies, problems analogies and pedagogical analogies (English, 2004). Problems analogies are two problems that have a common but thinking about the difference. Similarity can be a relational concept, settlement procedural steps, or others. Problems analogies consists of the target resource issues and problems. The source problem such as problems that are easy, can be done easily and does not use a lot of procedures. While the target problem in the form of a problem that a bit hard, difficult to work with, and using procedures quite a lot.

Solving problems by using analogical reasoning is enough to increase recent decades (Stavy and Tirosh 1993). Reasoner must admit the similarities in the relational structure between known problem (source problem) and new problem (target); is "structural alignment" or "mapping" between two problems that must be found (Supratman, Ryane & Rustina (2016),

Bassok 2002; Holyoak, Gentner, and Kokinov 2001). Problems are never used in the mathematical reasoning as used by English (2004) in the form of shaped comparison multiplication problems (source problem), "***Sarah has 52 books on her shelf. Sue has 4 times as many as Sarah. How many books Sue has?***" A comparison division problem had the same cover story, namely, "***Mary has 72 books on her shelf. This is 3 times as many as Peter has. How many books Peter has?***" Problems are designed to provide insight on the student's ability to see the nature of the initial problem to look more deeply at the underlying structural nature. After sorting, grouping, and troubleshooting the source, the children are introduced to some of the problems of the target. This problem has a similar structure to the source of the problem but it is more inclusive; namely beriri all the information needed to troubleshoot the source, plus some additional information (Reed, Ackinclose, & Voss, 1990). This meant that the child had to adapt or extend the source of solution procedure in order to use it to solve the target problem.

Beside, Assmus, Foster, & Fritzlar (2014) in their study wrote the problems analogies to the case arithmetic progression are "***Paul makes groups of counters on the table. Each new group contains more counters than the last group in a certain way. How many counters do you think he will put in the 20<sup>th</sup> group?***" (source problem) and "***Anna starts to read a book. She reads two pages on the first day. She continues to read the book, reading 2 pages more than the day before each day. How many pages will she have raed after 20 days in total?***" (target problem)". Problems analogies written Assmus et al (2014) have similarities in steps of completion of problems between source problems and target problems.

The most interseting part is the students ability to recognize similarities in structure and reason with this problem analogy to solve problems related to targets. English (2004) stated that the representation of students from the problem that often have a shortage of relational structures required proper reasoning by analogy, so that students do not just focus on the general nature of the surface of the problem. Even when the student demonstrates relational understanding, students tend to be spontaneous in using the analogy reason, if students do this, students often have difficulty in adapting the procedure source solution to meet the new requirements of the target problem (English, 2004). Several studies have shown how the subjects in the experimental situation tends to focus on the shallow nature while trying to use an analogy, while people in the context of non-experimental often use more structural nature of the reasoning analogy (Dunbar, 2001). Dunbar refers to this phenomenon as the "paradox analogical"; ie subjects require specialized training or assistance in analogy reasoning in research settings.they do not need assistance in using structural analogy in the context of neuralistic. Possible explanation for this paradox is the surface properties of experimental problems that can present a conceptual difficulties over the structure in nature than previously thought (Labato & Siebert, 2002). The work of Lobato (Lobato, 2003; Lobato & Siebert, 2002) shows how to transfer traditional research, which provides the subject using a similar task from the perspective of the researchers, can hide a lot of the learning process of the students. Researchers can gain insights into how individuals generate similarity between the problems of their own. Such insights can reveal how the new situation may be related to the previous picture of the situation of the students.

Traditional research on reasoning by analogy in solving the problem, it shows that learners require special knowledge base related to the use of analogy (English 2004). First, students should know the relational structure to generalize from the source or known issues, and if the problem sources should be taken out of memory, it should be done in terms of relational structures (Gentner and Gentner 1983; Gholson, Dattel, Morgan, and Eymard, 1989; Vosniadou 1989).

The ability to write your ideas mathematics or solving mathematical problems is needed by students. As a student of mathematics, the ability to complete the exercises, completing, or complete a math problem solving mathematically indispensable. What will be resolved and written by the students, will obviously involve other people to read it (Suyitno & Suyitni, 2015). The results of the answers to the exercises are done, it will probably be read friends of the class. Suyitno & suyitno (2015) added that results of the test will be read by the teacher, presents the solution of mathematical problems, will be heard by a friend of one class or a teacher.

Students present the solution of a mathematical problem solving. Solution written by a student if examined by a classmate or teacher can state that the solution can be written is a solution that is right or wrong. Mistake many students in mathematics may be caused by several factors. Comperehention less, language difficulties, anxiety, bustle and carelessness can be major factors in completion of tasks (Suyitno & Suyitno, 2015). Even the systematic errors is usually the consequence of misconceptions.

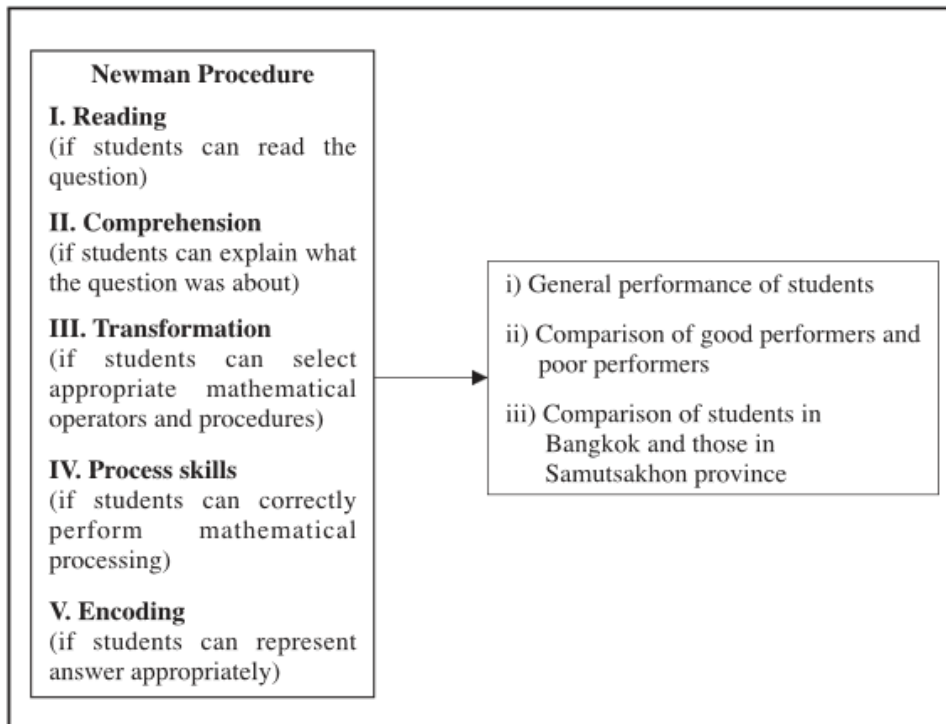
### **Newman Procedure**

Some many countries implement Newman procedures to determine the type of mistakes made by students in solving mathematical problems. To be able to solve mathematical problems (Dahlin & Watkins, 2000) says that understanding is more likely to lead to high quality results rather than memorization. Learning math is challenging, students are led to solve the problem very carefully. In the face of problems, students need the ability to identify and understand the problem if the problem at hand has similarities with the problems that have been solved. So that the concept or way of solving problem that has been used can be applied to the matter at hand. Besides, mathematics provides opportunities for students to develop mathematical abstract ideas that can improve the ability as a solver math problem.

Learning mathematics is deeply can make students do not make mistakes in solving math problems. And understanding of the material greatly assist students not much wrong. students need to build an understanding of understanding concepts, symbols, and mathematical theorem before trying to solve mathematical problems. Watkins & Biggs (2001) also did not agree that learning mathematics is dominated by memorization activities. Furthermore, they found that learning by memorization way may cause results not optimal.

We can find some of the mistakes made by students in solving mathematical problems. Various errors that can be found when students solving math problems. By using analyze procedures Newman, we can categorize the types of errors made by students in solving problems.

Based on the writings, Junaedi, Suyitno, Sugiharti, & Eng (2015), Suyitno & Suyitno (2015), White (2005), there are five types of errors according to Newman that caused errors students in solving mathematical problems. Five types of errors by Newman as follows.



1. Reading Error (R): Mistakes made in the resolution of problems classified as a reading error if students can not read key words or symbols written on the problem.
2. Comprehension Error (C): Students are not able to read all the words in question or a sentence about, but do not understand the overall meaning of the words so that students are not able to go further along the right channels for resolving problems.
3. Transformation Error (T): Students have been able to understand what the question will be searched completion, but will not be able to identify the operation or sequence of operations required to resolve the problem.
4. Process Skill Error (P): Students recognize the operation or sequence of operations, but did not know the procedures necessary to carry out the operation accurately.
5. Encoding Error (E): Students correctly solve the problem, but can not express the solution in the form of appropriate notation and can be accepted as a conclusion. Students are able to solve these problems, but in doing inference answer did not match the demand problem.

### **Purpose Of Research**

The purpose of this study was to analyze the errors that occurred in the students in solving problems analogies by procedure Newman with analogical reasoning.

### **Method**

This type of research is descriptive qualitative research. The collecting data used tests and interviews.

### **Participants**

The subjects of this research were 148 high school students. There are 93 students come from high school 1 Bima and 55 students come from high school 2 Mataram in West South Nusa, Indonesia.

## Materials

Research instrument is problems analogies (source problems and target problems). Problem analogies provided are “Find a solution to  $x^2 + 5x + 6 = 0$ ” (source problem) and “Find a solution to  $\cos^2 2x + 6 \sin x + 7 = 0$ ” (target problem).

## Procedures

Students are given the source of the problem (the problem of routine/simple matter) about the search for the roots of an equation. Once the source of the problem worked out by the student, then the student is given the target problem (the problem somewhat difficult/procedural problems). The students are given the source problem, which is routine and simple, such as finding a root of an equation. When the source problem is solved, the students get the target problem, which is more difficult than the source one. The source problem and target problem are different but they have similar concepts and solving operations.

## Results and Discussion

Almost all of the students answered correctly to the source problems. Students can't solve the target problems have in common the source problems with analogical reasoning. So that the students made many mistakes in solving the target problem. Based on the answers of students who obtained the target problems Newman procedural errors.

## Reading Errors

When students see and read about problem, the students do not understand the given problem (the problem of target). But try to understand target problems with working on the problem (see Figura 1, Figura 2(a) & 2(b)).

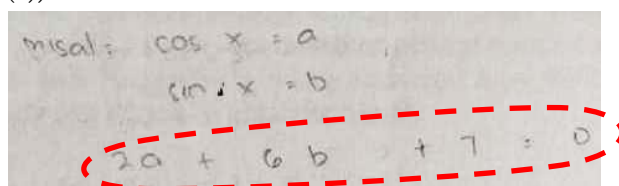
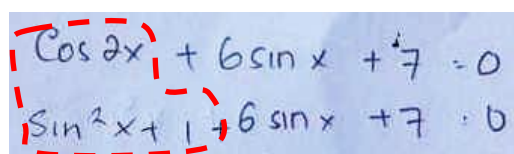
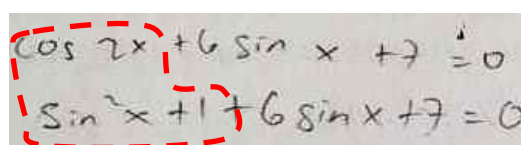


Figura 1 Reading errors

Based on Figura 1, students are able to read about and try to simplify the trigonometry problem into the algebra. Students perform analogy  $\cos x = a$  and  $\sin x = b$ , thereby forming the equation  $2a + 6b + 7 = 0$ . Furthermore, students  $\cos 2x = 2a$ .



(a)



(b)

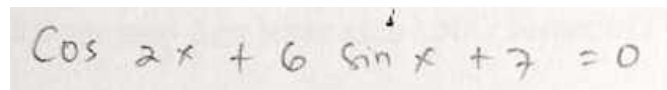
Figura 2. (a) & (b) Reading errors

As for this case from Figura 2(a) and Figura 2(b), students can read problems and trying to understand the problem in which he tried to find another form  $\cos 2x$ . Students write other forms of  $\cos 2x = \sin^2 x + 1$ . Errors that appear here are errors on the “+” which should “-”.

When interviews with students. For sources problem of student said very easy to do. But for target problem, students say: I do not know the answer to this problem. Further, I do not understand the problem. Learning materials trigonometric equations are difficult. Because student can not to change  $\cos 2x = 1 - 2\sin^2 x$  so that the student can't to solving the target problem.

### Comprehension errors

In comprehension errors, if students do not accurately transcribe what is known and questioned on the target problems facing (see Figura 3).



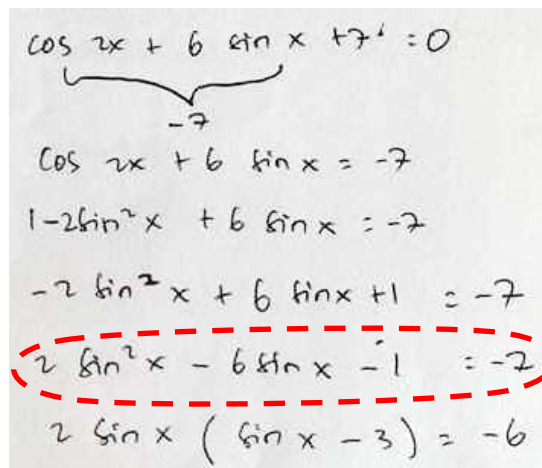
A photograph of a student's handwritten work on a piece of paper. The equation  $\cos 2x + 6 \sin x + 7 = 0$  is written in black ink. There is a small mark above the '7' in the original image.

Figura 3 Comprehension errors

When interviews with students, students say that students could rewrite what is known of the given problem but I am not understood about this problem.

### Transformation Errors

Errors occurred because the students are not transferring means of solving source problems to solve target problems. Students can not make trigonometric equations in the general form of a quadratic equation (see Figura 4).



A photograph of a student's handwritten work showing the transformation of the equation  $\cos 2x + 6 \sin x + 7 = 0$ . The steps are as follows:  
1.  $\cos 2x + 6 \sin x + 7 = 0$   
2. A bracket under the last two terms is labeled  $-7$ , leading to  $\cos 2x + 6 \sin x = -7$   
3.  $1 - 2\sin^2 x + 6 \sin x = -7$   
4.  $-2 \sin^2 x + 6 \sin x + 1 = -7$   
5.  $2 \sin^2 x - 6 \sin x - 1 = -2$  (This line is circled in red in the original image)  
6.  $2 \sin x (\sin x - 3) = -6$

Figura 4 Transformation errors

The results of the answers written by the students (Figura 4) have been able to read and understand the given problem. But students can not select and use a precise mathematical formula. In interview revealed that Students know the quadratic equation obtained equation and students trying to connect with the source of problems before. But students do not solve target problems by using the concept of completion quadratic equation/problem source.

### Skill Process Errors

The students did not solve the target problem with the appropriate mathematical procedure, which is the concept of quadratic equation used in solving the source problem (see Figura 5).

$$\begin{aligned} \cos 2x + 6 \sin x + 7 &= 0 \\ \cos 2x + 6 \sin x &= -7 \\ 1 - 2\sin^2 x + 6 \sin x &= -7 \\ -2 \sin^2 x + 6 \sin x + 1 &= -7 \\ 2 \sin^2 x - 6 \sin x - 1 &= -2 \\ 2 \sin x (\sin x - 3) &= -6 \\ \sin x &= \frac{-6}{2} \quad \vee \quad \sin x = -6+3 \\ \sin x &= -3 \quad \vee \quad \sin x = -3 \end{aligned}$$

Figura 5 Skill process errors

Based on Figura 5, students can write other forms of  $\cos 2x = 1 - 2 \sin^2 x$  there by forming quadratic equation  $2 \sin^2 x - 6 \sin x - 1 = -7$ . Students are trying to do factorization to find the roots of quadratic equations of trigonometry. However, students are not able to factor to determine the value  $\sin x$ .

From the results of the students' answers, the students do not form trigonometric equations in the general form quadratic equation  $ax^2 + bx + c = 0$ . Furthermore students perform settlement using factorization method on quadratic trigonometry equations, but wrong the results obtained. The results of interviews with students obtained:

- (1) Students are not using the settlement method quadratic equations in solving target problem like in solving source problems.
- (2) The students did not use the same arithmetic operation employed in solving the source problem.
- (3) The results of the factorization method incorrectly.

### Encoding Errors

The results obtained can not give a conclusion on the question asked. The students can not find the value of  $x$  in the form of degrees or radians (see Figura 6)

$$\begin{aligned} 2 \sin^2 x - 6 \sin x - 1 &= -2 \\ 2 \sin x (\sin x - 3) &= -6 \\ \sin x &= \frac{-6}{2} \quad \vee \quad \sin x = -6+3 \\ \sin x &= -3 \quad \vee \quad \sin x = -3 \end{aligned}$$

Figura 6 Encoding errors

Interviews with students, students find it difficult to change the value of  $x$  in the form of degrees or radians. Further, students can not find the degrees or radian that satisfies  $\sin x = -3$ .

### Careless Errors

Students perform such carelessness equating equation  $1 - 2 \sin x \times \sin x + 6 \sin x = -7$  with  $\times \sin x + 4 \sin x = -7$ ,  $\cos 2x$  equal  $\sin^2 x + 1$ ,  $\sin^2 x$  written  $2\sin^2 x$ , summing  $-2 \sin x$  in  $-2 \sin x \times \sin x$  with  $6 \sin x$  to be  $4 \sin x$ , etc (see Figura 7(a) & 7(b)).

$$\begin{aligned} \cos 2x + 6 \sin x + 7 &= 0 \\ 1 - 2\sin^2 x + 6 \sin x + 7 &= 0 \\ 1 - 2\sin^2 x + 6 \sin x &= -7 \\ 1 - 2\sin^2 x + \sin x + 6 \sin x &= -7 \\ 1 \sin x + 4 \sin x &= -7 \\ \sin x + 4 \sin x &= -7 \\ 5 \sin x &= -7 \\ \sin x &= \frac{-7}{5} \end{aligned}$$

(a)

$$\begin{aligned} \cos 2x + 6 \sin x + 7 &= 0 \\ \sin^2 x + 1 + 6 \sin x + 7 &= 0 \\ \sin^2 x + 6 \sin x + 8 &= 0 \\ 2 \sin^2 x &= \frac{-8}{6} \\ \sin^2 x &= -\frac{4}{3} \times \frac{1}{2} \\ \sin^2 x &= -\frac{4}{6} \\ \sin x &= -\frac{1}{2} \sqrt{3} \end{aligned}$$

(b)

Figura 7 (a) &amp; (b) Careless errors

### Errors Analogical Reasoning

Errors that appear in solving the problems analogies based Newman procedure has similarities with results studi conducted by Suyitno & Suyitno (2015). in general, students often make mistakes in understanding the problem. students do not know what is known of the problem and what was asked in the problem. The similar errors is obtained in this study are reading errors, comprehension errors, transformation errors, skill process errors, encoding errors, and careless errors (Junaedi, Suyitno, Sugiharti, and Eng, 2015; Suyitno & Suyitno, 2015).

By using analogical reasoning, students can solve the target problems have in common with the source problems. Based-on instruments provided students, analogical reasoning students do after structuring. Students perform encoding and inferring process that aims to shape the same problem as the source problems. Futhermore student can do the mapping to find the relationship between the target problems and the source problems. Students can perform operations on the target problems such as the source problems. The results obtained from the students can do justification and response to the target problems. Based-on this research, instrument problems analogies given to students emergence position teorities analogical reasoning is students perform mapping and applying without inferring.

### Conclusion

Based on the analysis of research data, there are several conclusions that can be obtained.

1. Errors that appear in this study there are 6 types of errors are five types of errors based procedures Newman and one types of errors resulting from carelessness students.
  - a. Reading errors is student can not to change  $\cos 2x = 1 - 2\sin^2 x$  so that the student can't to solving the target problem
  - b. Comprehension errors is students do not accurately transcribe what is known and questioned on the target problems facing.
  - c. Transformation errors is students can not make trigonometric equations in the general form of a quadratic equation
  - d. Skill process errors is students are not solve target problems by using the concept of completion quadratic equation (source problem) with the appropriate mathematical procedures.
  - e. Encoding errors is student can not give a conclusion on the question asked. Because students can not find the value of x in the form of degrees or radians.
  - f. Careless errors is students perform such carelessness equating equation  $1 - 2 \sin x \times \sin x + 6 \sin x = -7$  with  $\times \sin x + 4 \sin x = -7$ ,  $\cos 2x$  equal  $\sin^2 x + 1$ ,  $\sin^2 x$  written  $2\sin^2 x$ , summing  $-2 \sin x$  in  $-2 \sin x \times \sin x$  with  $6 \sin x$  to be  $4 \sin x$ , etc.



2. Concept material has similarities with problems analogies that is being done needs to be given greater depth in previous learning process.
3. Material concept has similarities with problems analogies that are being solved should be given more depth in the learning process before.

Based on this study provides preliminary findings that in solving the problems analogies, students can make mistakes. This case provides an opportunity to do more research on how the process of the occurrence of errors in analogical reasoning.

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